

المحاضرة الاولى

التفاضل

e n g m a o 9 7

Second Term
Calculus

Lecture 1

- المميز
الزائفة
Hyberbolic Functions , Inverse Functions.
- Integrals.
- Applications of Integrals.

⇒ Math : 150 marks → 100 marks Final
50 marks Term work "25 Algebra"
25 calculus

1 Hyperbolic Functions

⇒ * $\frac{e^x - e^{-x}}{2} = \sinh x$ ^{hyber sin} , $\frac{2}{e^x - e^{-x}} = \frac{1}{\sinh x} = \operatorname{csch} x$

* $\frac{e^x + e^{-x}}{2} = \cosh x$ ^{hyber cos} , $\frac{2}{e^x + e^{-x}} = \frac{1}{\cosh x} = \operatorname{sech} x$

* $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, $\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

#engmao97

⇒

$$y = F(u)$$

$$y' = F'(u) \cdot u'$$

$$\sinh u(x)$$

$$\cosh u(x) \cdot u'(x)$$

$$\cosh u(x)$$

$$\sinh u(x) \cdot u'(x)$$

$$\tanh u(x)$$

$$\operatorname{sech}^2 u(x) \cdot u'(x)$$

$$\coth u(x)$$

$$-\operatorname{csch}^2 u(x) \cdot u'(x)$$

$$\operatorname{sech} u(x)$$

$$-\operatorname{sech} u(x) \cdot \tanh u(x) \cdot u'(x)$$

$$\operatorname{csch} u(x)$$

$$-\operatorname{csch} u(x) \cdot \coth u(x) \cdot u'(x)$$

⇒ Find y'

$$(1) \quad y = x^3 + \sinh x^2$$

$$y' = 3x^2 + \cosh x^2 \cdot 2x$$

$$(2) \quad y = \ln x \cdot \tanh x (3x+1)$$

$$y' = \frac{1}{x} \cdot \tanh(3x+1) + \ln x \cdot \operatorname{sech}^2(3x+1) \cdot 3$$

$$(3) \quad y = \cosh^3 2x + \sin 2x \cdot \operatorname{sech} 3x$$

$$y' = 3(\cosh 2x)^2 \cdot \sinh 2x \cdot 2 +$$

$$\cos 2x \cdot 2 \cdot \operatorname{sech} 3x + \sin 2x \cdot -\operatorname{sech} 3x \cdot$$

$$\tan 3x \cdot 3$$

#engmao97

2 Inverse Functions

$$\Rightarrow y = F(x) = 2 + \frac{1}{x}$$

x	y
-1	1
1	3
2	$\frac{5}{2}$
3	$\frac{7}{3}$

$$\Rightarrow y - 2 = \frac{1}{x}$$

$$x = \frac{1}{y-2} \quad , \quad F^{-1}(x) = \frac{1}{x-2}$$

\Rightarrow If $F(x)$ is Function then its inverse is denoted by $F^{-1}(x)$ and it can be obtained by obtaining x in terms of y .

\Rightarrow Trigonometric Functions : , Inverse Trigonometric Functions

$$\sin x$$

$$\sin^{-1} x = \arcsin x$$

$$\cos x$$

$$\cos^{-1} x$$

$$\tan x$$

$$\tan^{-1} x$$

$$\cot x$$

$$\cot^{-1} x$$

$$\sec x$$

$$\sec^{-1} x$$

$$\csc x$$

$$\csc^{-1} x$$

#engmao97

⇒ Hyperbolic Functions, Inverse Hyperbolic Functions

$$\sinh x$$

$$\sinh^{-1} x$$

$$\cosh x$$

$$\cosh^{-1} x$$

$$\tanh x$$

$$\tanh^{-1} x$$

$$\coth x$$

$$\coth^{-1} x$$

$$\operatorname{sech} x$$

$$\operatorname{sech}^{-1} x$$

$$\operatorname{csch} x$$

$$\operatorname{csch}^{-1} x$$

$$y = F(x)$$

$$y = F^{-1}(x)$$

$$\sin^{-1} u(x)$$

$$\frac{1}{\sqrt{1-u^2}} \cdot u'(x)$$

$$\cos^{-1} u(x)$$

$$\frac{-1}{\sqrt{1-u^2}} \cdot u'(x)$$

$$\tan^{-1} u(x)$$

$$\frac{1}{1+u^2} \cdot u'(x)$$

$$\sinh^{-1} u(x)$$

$$\frac{1}{\sqrt{1+u^2}} \cdot u'(x)$$

$$\cosh^{-1} u(x)$$

$$\frac{1}{\sqrt{u^2-1}} \cdot u'(x)$$

$$\tanh^{-1} u(x)$$

$$\frac{1}{1-u^2} \cdot u'(x)$$

$$\Rightarrow \sin^{-1} x \neq (\sin x)^{-1}$$

$$\sin^{-2} x = (\sin x)^{-2} = \frac{1}{\sin^2 x}$$

$$(\sin^{-2} x)^3 = \sin^{-6} x$$

$$(\sin^{-1} x)^3 \neq \sin^{-3} x$$

#engmao97

⇒ Find y'

(1) $y = \sin^{-1} 3x + \sin^{-2} 3x$

$$y' = \frac{1}{\sqrt{1-(3x)^2}} \cdot 3 + -2 \cdot (\sin 3x)^{-3} \cdot \cos 3x \cdot 3$$

(2) $y = (\sinh^{-1} 2x)^3 + \tanh^{-3} x$

$$y' = 3 (\sinh^{-1} 2x)^2 \cdot \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 + -3 (\tanh x)^{-4} \cdot \operatorname{sech}^2 x$$

⇒ Show that $\sinh^{-1} x = \ln(x + \sqrt{1+x^2})$ and it is odd Function (proof)

$$y = \sinh^{-1} x$$

$$x = \sinh y = \frac{e^y - e^{-y}}{2}$$

$$2x = e^y - e^{-y} \quad \times e^y$$

$$2x \cdot e^y = e^{2y} - 1$$

$$e^{2y} - 2x \cdot e^y - 1 = 0$$

$$(e^y)^2 - 2x(e^y) - 1 = 0$$

$$Ax^2 + Bx + C = 0$$

$$x = e^y$$

$$A = 1, B = -2x, C = -1$$

$$\therefore x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\therefore e^y = \frac{2x \pm \sqrt{4x^2 - 4(1)(-1)}}{2(1)} = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = \frac{2x \pm 2\sqrt{x^2 + 1}}{2}$$

$$= x \pm \sqrt{x^2 + 1}$$

$$\therefore e^y = x + \sqrt{x^2 + 1}$$

#engmao97

$$\therefore y = \ln(x + \sqrt{1+x^2})$$

$$\therefore y = \sinh^{-1} x$$

$$\therefore \sinh^{-1} x = \ln(x + \sqrt{1+x^2})$$

#

$$\sinh^{-1}(-x) = \ln(-x + \sqrt{1+(-x)^2})$$

$$\sinh^{-1}(-x) = \ln(-x + \sqrt{1+x^2})$$

$$\sinh^{-1}(-x) = \ln\left(-x + \sqrt{1+x^2} \cdot \frac{-x - \sqrt{1+x^2}}{-x - \sqrt{1+x^2}}\right)$$

$$\sinh^{-1}(-x) = \ln\left(\frac{x^2 - (x^2+1)}{-x - \sqrt{1+x^2}}\right)$$

$$\sinh^{-1}(-x) = \ln\left(\frac{-1}{-x - \sqrt{1+x^2}}\right)$$

$$\sinh^{-1}(-x) = \ln\left(\frac{1}{x + \sqrt{1+x^2}}\right)$$

$$\sinh^{-1}(-x) = \underbrace{\ln 1}_{=0} - \ln(x + \sqrt{1+x^2})$$

$$\sinh^{-1}(-x) = -\ln(x + \sqrt{1+x^2})$$

$$\therefore \sinh^{-1}(x) = \ln(x + \sqrt{1+x^2}) \text{ is odd Function}$$

#

#engmao97